SSB-UDL - deflection *vs* stress

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For "Euler-Bernoulli beams"; derive for a symmetrical (about the major axis) simply-supported-beam
with
uniformly-distributed-load
(ssb-udl)
the relationship between deflection and stress.
This is the maximum deflection of that beam at the mid-span, at the maximum stress in that beam also occurring at the mid-span.

Symbols used

are those familiar in beam equations:

I = Second Moment of Area
Z = Section Modulus
M = a Moment; a bending and/or turning force
w = force per unit length along the beam (in Newtons/metre; N/m)
L = length of the beam between supports
H = height of symmetrical section in direction it is being bent
σ = stress (in N/m^2)
y = beam bending deflection transverse to the length L dimension

Familiar equations for ssb-udl

Deflection:

$$y = \frac{5wL^4}{384EI}$$

Maximum Moment:

$$M = \frac{wL^2}{8}$$

Always-applying beam equations

$$M = \sigma Z$$

For beam symmetrical about major axis regarding load and deflection

$$Z = \frac{2I}{H} \to M = \frac{2\sigma I}{H}$$

Derivation

$$y = \frac{5wL^4}{384EI}$$

$$M = \frac{wL^2}{8} \to w = \frac{8M}{L^2}$$

$$y = \frac{5}{384}\frac{8M}{L^2}\frac{L^4}{EI} = \frac{40}{384}\frac{ML^2}{EI}$$

$$y = \frac{40}{384}\frac{2\sigma I}{H}\frac{L^2}{EI} = \frac{80}{384}\frac{\sigma L^2}{EH} = \frac{5}{24}\frac{\sigma L^2}{EH}$$

The derived equation:

Into the deflection *vs* force-per-unit-length equation substituting the Moment equation and the fundamental $M = \sigma Z$ beam equation gives the deflection *vs* **stress** equation:

$$y = \frac{5}{24}\frac{\sigma L^2}{EH}$$

The Second Moment of Area I has cross-cancelled during the derivation. Giving this unusual specific case of a beam equation which does not contain or need I.
The practical significance is that the full cross-sectional specification in the design information needed to calculate I is unnecessary when applying this equation to analyse beams.

ssb-udl *vs* ssb-cl

The deflection *vs* stress equation for a ssb-cl; a simply supported beam centrally-loaded - has elsewhere been derived as

$$y_{\mathrm{ssb-cl}} = \frac{\sigma L^2}{6EH} \rightarrow y_{\mathrm{ssb-cl}} = \frac{4}{24}\frac{\sigma L^2}{EH}$$

This is 1/4 part different from

$$y_{\mathrm{ssb-udl}} = \frac{5}{24}\frac{\sigma L^2}{EH}$$